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# Multistep direct emission from nucleon-induced reactions

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**Abstract.** Unbound 1p1h states are excited together with the bound ones in one-step direct processes induced by nucleons of energy greater than the particle binding energy. The cross-sections of the one-step direct reactions to bound final states are folded into a convolution integral to obtain the multistep cross-sections. This is done in the framework of the theory of Feshbach, Kerman and Koonin (FKK) that describes the emission of one particle. The processes to unbound 1p1h final states give rise to more complicated direct reactions, that are evaluated by an approximate method. The comparison of the theory with inclusive proton inelastic scattering by iron and with the charge exchange (p,n) reaction on zirconium is revised, at energies from 25 MeV to 120 MeV. It is shown that in the case of the (p,n) reaction the theory of FKK does not account for approximately 35% of the direct processes.

**PACS.** 25.40.Kv Charge-exchange reactions – 25.40.Ep Inelastic proton scattering – 24.60.Gv Statistical multistep direct reactions

## 1 Introduction

The convolution integral of the theory of Feshbach, Kerman and Koonin [1] makes the calculation of cross-sections for the multistep direct (MSD) reactions feasible and requires as input the cross-sections for excitation of well-defined final states. These can be either collective one-phonon vibrations and/or one-step excitations of incoherent particle-hole pairs, both satisfying the energy-weighted sum rules (EWSRs) [2,3]. The incoherent particle-hole states do not pose any problems at low energies even when unbound. In the latter case it is safe to assume that the low-energy unbound particle undergoes mainly damping transitions, before ending in the final quasi-bound state in the continuum. However, for nucleon-induced reactions at incident energies above the potential well depth, *i.e.* above 40 MeV, the nucleus is likely to be excited to unbound particle-hole final states that require special attention [4]. In such a case the one-step process leaves two quasi-free particles in the continuum, namely the leading particle and the excited particle in the final unbound state. Both particles are emitted giving rise to a knockout reaction, (a,ab). The transition matrix involves a three-body final state that can be treated only very approximately in the usual DWBA. Furthermore, the knockout reaction can be followed by multistep scattering processes. So far, the latter have been crudely evaluated in the framework of the FKK theory as primary emission associated with the direct emission of a second particle (MSD2) [5]. However, in the current approximate method of [5], only the simultaneous emission of two particles obtained from the first stage of the reaction (1SD2) can be described reliably, since the emission of two particles from higher stages requires the use of $MSD_{unbound}$ cross-sections to unbound final states. Only the very specific ones of such $MSD_{unbound}$ cross-sections can be calculated in the framework of the FKK formalism (see sect. 2). Rigorously such contributions can be estimated in the framework of the theory of the multistep two-particle emission of Ciangaru [6], who has applied similar statistical postulates as those adopted by FKK. However, the resulting formalism is rather involved and has never been implemented to its full extent in practical calculations [7]. On the other hand, measurements of the emission of two protons indicate that, in addition to pure knockout to discrete states of the final nucleus, a large part of the reactions corresponds to events which have clearly undergone further rescattering, subsequent to an initial collision between the projectile and a nucleon bound in the target [7,8].

In the calculations reviewed in the following section the one-step direct (1SD) reaction cross-sections of [3] are used. They include a macroscopic DWBA term describing the collective vibrations (vib) and the microscopic DWBA terms describing the incoherent particle-hole excitations (ph). The density of the 1p1h states with restrictions on

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the particle binding and the finite hole depth [9] is used to determine the (ph) terms of the $1SD_{bound}$ cross-section to bound final states, while the density of the 1p1h states given only by the hole depth restriction [10] is used to obtain the (ph) terms of the 1SD cross-section to both the bound and the unbound final states. The $1SD_{bound}$ includes the incoherent cross-sections to bound particle-hole states as well as the ones to one-phonon, collective vibrations, and thus it describes the emission of one particle. Since the theory of FKK allows only for one particle in the continuum, it is the $1SD_{bound}$ cross-sections that are folded in the convolution integral to give the $MSD_{bound}$ cross-sections of FKK. The use of the EWSRs to control the cross-sections of the one-step reactions in the folding integral also requires the use of $1SD_{bound}$ cross-sections. On the other hand, the $1SD_{unbound}$ cross-section to unbound final particle-hole states is related to more complicated direct processes, *e.g.* it describes one of two emitted particles. The emission of the other one of the two, 1SD2, is evaluated from the 1SD cross-section according to the prescription of [5]. The $1SD_{unbound}$, integrated over angle and energy, provides then the total cross-section for the emission of two particles. The continuum particle in the unbound final state of the $1SD_{unbound}$ can either escape the nucleus, giving rise to a one-step two-particle emission, or it can undergo one or a few collisions before being emitted in a multistep two-particle process. Thus, the rescattering events, observed in the experiments cited above, are included in $1SD_{unbound}$. Only the simultaneous emission of the two particles is considered since the alternative, successive emission of a secondary particle after one or a few rescattering collisions, subsequent to a primary emission, was found to be relatively unimportant [5,6,11,12].

By distinguishing between a proton or neutron leading particle in the continuum, a number of different sequences of reaction stages can contribute to a given MSD reaction. All the different sequences are calculated herein. A full theoretical interpretation of the experimental data at incident energies ranging from 25 to 120 MeV is presented in the following sections.

## 2 Model calculations and comparison with experiments

The $MSD_{bound}$ cross-section of FKK is obtained by multiple folding of the one-step cross-sections to bound final states [1,13]:

$$\begin{aligned} MSD_{FKK} = MSD_{bound} = \int \frac{m_1 E_1}{(2\pi)^2\hbar^2} dE_1 d\Omega_1 \dots \\ \times \int \frac{m_{M-1}E_{M-1}}{(2\pi)^2\hbar^2} dE_{M-1} d\Omega_{M-1} \\ \times (1SD_{bound})_M \times S^{-2} (1SD_{bound})_{M-1} \cdots \\ \times S^{-2} (1SD_{bound})_1 . \end{aligned} \quad (1)$$

In eq. (1) $M-1$ of the successive double-differential one-step cross-sections include non-DWBA matrix elements with biorthogonally conjugated incoming distorted waves $\langle \hat{\chi}^{(+)} |$. The non-DWBA matrix elements can be expressed in terms of normal DWBA matrix elements by using the relation $\langle \hat{\chi}^{(+)} | = S^{-1} \langle \chi^{(-)} |$ [14]. Since for each transfer of multipolarity or orbital angular momentum $L$ a number of partial waves $\ell_{M-1}$, $\ell_{M-2}$ contribute to the incoming and outgoing distorted waves, respectively, in the transition matrix elements, each of the incoming waves in the outgoing channel is enhanced by the corresponding $S^{-1}_{\ell_{M-1}}$. On the other hand, in the statistical theory of FKK the enhanced transition matrix elements are averaged over energy. The overall effect of the energy average of the enhanced matrix elements for a specific $L$, is approximated by the effect of an average enhancing factor $\langle S_L \rangle^{-1}$ acting on the partial waves $\ell_{M-1}$. The elastic scattering matrix elements $S_L$ are expressed by the partial-wave transmission coefficients $T_L$'s of the optical potential, $S_L^2 = 1 - T_L$. The energy averaging $\langle S_L \rangle^2 = 1 - \langle T_L \rangle$ ensures that the average enhancement is smooth and free of any fluctuations or singularities that could arise at the energies of the single-particle resonances where $T_L \approx 1$. The impact of the average enhancing factors $\langle S_L \rangle^{-1}$ is smaller than that of the $S^{-1}_{\ell_{M-1}}$ ones [3] and convergent $MSD_{bound}$ cross-sections are obtained [4,15–18].

The double-differential, enhanced $S^{-2}(1SD_{bound})$ cross-sections in (1), are expressed as sums over $L$, and consist of the collective term $(\mathrm{vib}S^{-2}) = \Sigma_{L<5}\sigma_L(\mathrm{vib})\langle S_L \rangle^{-2}$ and of the term describing the incoherent excitation of the particle-hole pairs $(\mathrm{ph}S^{-2}) = \Sigma_{L>4}\sigma_L(\mathrm{ph})\langle S_L \rangle^{-2}$ [3]. Thus, the $MSD_{bound}$ cross-sections of eq. (1) contain the following combinations of the two terms [15,16]:

1SD, (vib) + (ph),

2SD, $(\mathrm{vib}S^{-2}, \mathrm{vib}) + (\mathrm{ph}S^{-2}, \mathrm{vib}) + (\mathrm{vib}S^{-2}, \mathrm{ph})$
$+ (\mathrm{ph}S^{-2}, \mathrm{ph})$,

etc.

The macroscopic DWBA cross-sections in (vib) include form factors given by a derivative of the deformed optical potential and are summed over the final known one-phonon $2^+$, $3^-$ and $4^+$ states in the even-even core nuclei of interest. The strength is determined by the phenomenological deformation parameters $\beta_L$. The dipole, quadrupole and the low-energy component of the octupole (LEOR) giant resonances are also taken into account. The strength parameters for the giant resonances are obtained by depletion of the EWSRs by the strengths of the known one-phonon levels. The weak-coupling model is applied for odd-mass nuclei. The isovector (vib) cross-section in case of the charge exchange (p,n) reaction is calculated in an approximate way described in detail in [4,17].

The microscopic DWBA cross-sections in (ph) include the two-particle form factor with real effective Yukawa interaction of strength $V_{\pi,\pi} = V_{\nu,\nu} = 12.7$ MeV and $V_{\pi,\nu} = V_{\nu,\pi} = 43.1$ MeV [19]. The decrease of the interaction strength with increasing energy [20] is accounted for. The microscopic DWBA cross-sections are averaged over final, shell model particle-hole states $(j_\mathrm{p} j_\mathrm{h}^{-1})_{LM}$. In case of highly excited final states, where particle excitations

**Table 1.** The multistep sequences of direct reaction stages that contribute to the $^{90}$Zr(p,n)$^{90}$Nb reaction at 120 MeV obtained with the non-DWBA matrix elements.

| (1SD$_{\text{bound}}$) (mb) | (2SD$_{\text{bound}}$) (mb) | (3SD$_{\text{bound}}$) (mb) | (4SD$_{\text{bound}}$) (mb) |
|---|---|---|---|
| (pn) 24 | (pp$S^{-2}$, pn) 14 | (pp$S^{-2}$, pp$S^{-2}$, pn) 13.7 | (pp$S^{-2}$, pp$S^{-2}$, pp$S^{-2}$, pn) 12.9 |
| | (pn$S^{-2}$, nn) 7 | (pp$S^{-2}$, pn$S^{-2}$, nn) 5.4 | (pp$S^{-2}$, pp$S^{-2}$, pn$S^{-2}$, nn) 8.3 |
| | | (pn$S^{-2}$, np$S^{-2}$, pn) 3.3 | (pp$S^{-2}$, pn$S^{-2}$, nn$S^{-2}$, nn) 3.5 |
| | | (pn$S^{-2}$, nn$S^{-2}$, nn) 2.6 | (pn$S^{-2}$, np$S^{-2}$, pp$S^{-2}$, pn) 3.2 |
| | | | (pp$S^{-2}$, pn$S^{-2}$, np$S^{-2}$, pn) 2.6 |
| | | | (pn$S^{-2}$, np$S^{-2}$, pn$S^{-2}$, nn) 2.0 |
| | | | (pn$S^{-2}$, nn$S^{-2}$, nn$S^{-2}$, nn) 1.6 |
| | | | (pn$S^{-2}$, nn$S^{-2}$, np$S^{-2}$, pn) 1.2 |
| Total 24 | 21 | 25 | 35 |
| | (5SD$_{\text{bound}}$) (mb) | (6SD$_{\text{bound}}$) (mb) | (7SD$_{\text{bound}}$) (mb) |
| Total | 52 | 65 | 58 |

may be in the continuum, the latter are made quasi-bound. This then allows the use of a standard DWBA code such as DWUCK-4 [21] to determine approximate cross-sections to final unbound particle-hole states. The spectroscopic amplitude $(2j_{\text{h}}+1)^{1/2}$ for the microscopic option of DWUCK-4 is used. The macroscopic cross-sections are also calculated with the DWUCK-4 code. The optical potentials of [22–24] are used for neutrons and those of [24, 25] for protons. The same potentials are used for calculating the enhancement of the MSD$_{\text{bound}}$ cross-sections.

The microscopic (ph) terms of the 1SD$_{\text{bound}}$ cross-section are calculated with densities of the final 1p1h states given by the two-component formula of [9], with particles restricted to energies below the binding energy $B_i$ and the holes restricted by the potential depth $\epsilon_{\text{F}}^j$,

$$\begin{aligned}\omega_{1\text{p}_i1\text{h}_j}(U,B_i,\epsilon_{\text{F}}^j) = g_ig_j[U-(U-B_i)\Theta(U-B_i)\\ -(U-\epsilon_{\text{F}}^j)\Theta(U-\epsilon_{\text{F}}^j)\\ +(U-B_i-\epsilon_{\text{F}}^j)\Theta(U-B_i-\epsilon_{\text{F}}^j)],\end{aligned} \tag{2}$$

and with $i=j=\pi$ for $\text{p}_\pi=\text{h}_\pi=1$ and $\text{p}_\nu=\text{h}_\nu=0$; $i=j=\nu$ for $\text{p}_\nu=\text{h}_\nu=1$ and $\text{p}_\pi=\text{h}_\pi=0$; $i=\nu$, $j=\pi$ for $\text{p}_\nu=\text{h}_\pi=1$ and $\text{p}_\pi=\text{h}_\nu=0$; $i=\pi$, $j=\nu$ for $\text{p}_\pi=\text{h}_\nu=1$ and $\text{p}_\nu=\text{h}_\pi=0$; with $\Theta$ being the Heaviside step function and the equidistant single-particle state densities for protons and neutrons, $g_\pi=Z/13$, $g_\nu=N/13$, respectively.

The (vib) and the (ph) terms are then lumped together to give the 1SD$_{\text{bound}}$ cross-section for a single reaction stage (nn), (np), (pp) and (pn). This allows us then to distinguish only between the different sequences of the above-mentioned reaction stages that result from the $M$ collisions of the leading continuum nucleons with the nucleons of the target. Equation (1) is used so many times as different sequences contribute to the MSD$_{\text{bound}}$ cross-section in question. The sequences that contribute to the (p,n) reaction are listed in table 1 and the ones that contribute to the (p,p$'$) reaction are the following [4,18]:

1SD, (pp),

2SD, (pp$S^{-2}$, pp) + (pn$S^{-2}$, np),

3SD, (pp$S^{-2}$, pp$S^{-2}$, pp) + (pp$S^{-2}$, pn$S^{-2}$, np) + (pn$S^{-2}$, nn$S^{-2}$, np) + (pn$S^{-2}$, np$S^{-2}$, pp),

etc.

It is shown in [27] that in case of neutron scattering the sequences of reaction stages including only neutrons

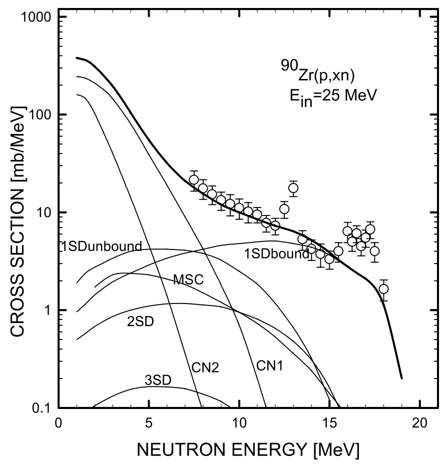


**Fig. 1.** Comparison of the enhanced MSD = MSD$_{\text{bound}}$ cross-sections calculated with the non-DWBA matrix elements with the inclusive spectrum of neutrons emitted from the $^{90}$Zr(p,$x$n)$^{90}$Nb reaction, at incident proton energy of 25 MeV [26]. The thick line is a sum of all contributions. CN1 and CN2 denote primary and secondary neutrons evaporated from the compound nucleus. MSC shows the pre-equilibrium compound emission. The 1SD cross-section is split into the 1SD$_{\text{bound}}$ and 1SD$_{\text{unbound}}$ components. The 2SD and 3SD cross-sections are obtained by folding 1SD$_{\text{bound}}$. The experimental peaks are due to the isobaric analog and the low-energy neutron-hole states [17].

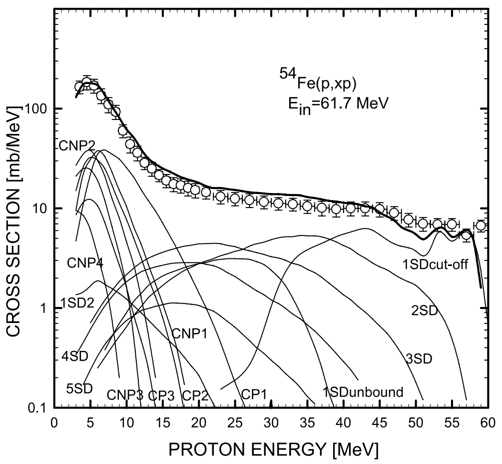


**Fig. 2.** Comparison of the enhanced MSD = $\text{MSD}_{\text{bound}}$ cross-sections calculated with the non-DWBA matrix elements with the inclusive spectrum of protons from the $^{54}$Fe(p,$x$p)$^{54}$Fe reaction, at incident energy of 61.7 MeV [28]. CP1-3 denote primary to tertiary protons evaporated from the compound nucleus, respectively. CNP1-4 denote protons preceded by evaporation of a neutron. Direct emission of two particles is given by $1\text{SD}_{\text{unbound}}$ + 1SD2. The 2SD to 5SD cross-sections are obtained by folding $1\text{SD}_{\text{cut-off}}$. The $1\text{SD}_{\text{cut-off}}$ cross-section approximates the particle binding restriction in the final 1p1h state density [4].

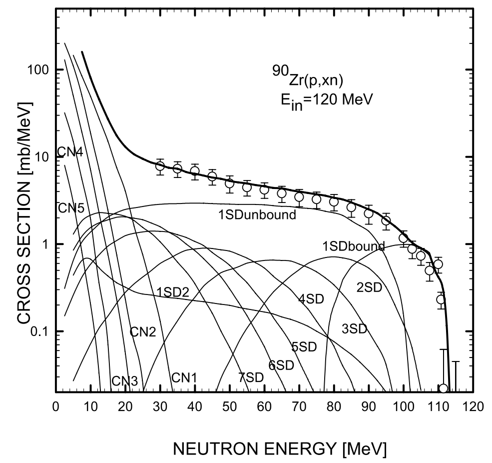


**Fig. 3.** The same as in fig. 1 but for the incident energy of 120 MeV [29]. Direct emission of two particles is given by $1\text{SD}_{\text{unbound}}$ + 1SD2 [17].

dominate markedly over the other ones including also protons. However, in the case of proton scattering or charge exchange reactions one cannot foresee if and which sequences can be neglected. Therefore, all sequences of reaction stages are calculated and summed in order to obtain the $\text{MSD}_{\text{bound}}$ cross-sections compared with experiments in figs. 1 to 3. The cut-off of the low-energy portion of the $1\text{SD}_{\text{bound}}$ spectra, in figs. 2 and 3, is a direct result of considering only bound final 1p1h states. The $1\text{SD}_{\text{bound}}$ cross-sections observe the EWSRs independent on the incident energy.

The 1SD cross-section to both the bound and unbound 1p1h states is calculated with the two-component expression for the level density, restricted on the hole depth only [10],

$$\omega_{1\text{p}_i 1\text{h}_j}(U,\epsilon_{\text{F}}^j) = g_i g_j [U - (U - \epsilon_{\text{F}}^j)\Theta(U - \epsilon_{\text{F}}^j)] \,. \qquad (3)$$

The indexes $i$ and $j$ are the same as in eq. (2).

The $1\text{SD}_{\text{unbound}}$ to unbound final states, in figs. 1 to 3, is obtained by subtracting $1\text{SD}_{\text{bound}}$ from 1SD, *i.e.* $1\text{SD}_{\text{unbound}} = 1\text{SD} - 1\text{SD}_{\text{bound}}$. At incident energies lower than the potential well depth, *i.e.* below 40 MeV, $1\text{SD}_{\text{unbound}}$ is that part of the 1SD cross-section that gives rise to the emission of one particle, followed by one or a few damping transitions of the low-energy unbound particle until the final quasi-bound state embedded in the continuum is reached. Such multistep reactions, that damp a considerable portion of the direct reaction flux into the compound-nucleus states beyond the 2p1h doorway-state, give rise to gradual absorption [30,31].

Some contributions to the gradual absorption from damping that follows the multistep emission of one particle and/or to the multistep two-particle emission can be estimated from the $\text{MSD}_{\text{unbound}}$ obtained by generalization of the 1SD case (above), using $\text{MSD}_{\text{unbound}} = \text{MSD} - \text{MSD}_{\text{bound}}$. The MSD cross-section here involves the convolution of $(M-1)$ successive, enhanced $S^{-2}(1\text{SD}_{\text{bound}})$ cross-sections,

$$\begin{aligned}\text{MSD} = &\int \frac{m_1 E_1}{(2\pi)^2\hbar^2}\text{d}E_1\text{d}\Omega_1 \ldots \\ &\times \int \frac{m_{M-1}E_{M-1}}{(2\pi)^2\hbar^2}\text{d}E_{M-1}\text{d}\Omega_{M-1} \\ &\times (1\text{SD})_M \times S^{-2}\,(1\text{SD}_{\text{bound}})_{M-1}\cdots \\ &\times S^{-2}\,(1\text{SD}_{\text{bound}})_1 \,. \end{aligned} \qquad (4)$$

Contrary to the $(1\text{SD}_{\text{bound}})_M$, in eq. (1), the unenhanced cross-section of the last, $M$-th reaction step $1\text{SD}_M$ in (4), is not restricted by particle binding and includes contributions to both the bound and unbound final states,

$$1\text{SD}_M = \left(\frac{\text{d}^2\sigma}{\text{d}E_M\text{d}\Omega_M}\right)^{\text{DWBA}}_{1\text{SD}} . \qquad (5)$$

However, one has to emphasize that the damping transitions of the low-energy unbound particle that follow the one-step or multistep emission, considered above, are not the same as those discussed in [30,31]. In the papers just cited the multistep transitions including damping precede any emission and therefore, they end up with the formation of the $(n+1)$p$n$h quasi-bound compound states of the composite $(A+1)$-nucleus, whereas we consider damping into the $n$p$n$h compound-nucleus states in the $A$-nucleus. Both processes contribute at $n > 1$ to the gradual absorption.

At energies higher than 40 MeV, $1\text{SD}_{\text{unbound}}$ is that part of the 1SD cross-section that gives rise to the emission of two particles since the high-energy particle of the unbound final state easily escapes the nucleus even after a few rescattering collisions with its bound nucleons.

**Table 2.** The integrated cross-sections for the $^{90}Zr(p,n)^{90}Nb$ reaction including enhanced $MSD_{bound}$ contributions calculated with non-DWBA matrix elements. All cross-sections are in mb. The cross-sections are verified by comparison with experimental data, as in figs. 1 and 3.

| Incident energy | 25 MeV | 45 MeV | 80 MeV | 120 MeV | |
|---|---|---|---|---|---|
| Reaction | | | | | Emission |
| $1SD_{bound}$ | 60 | 57 | 34 | 24 | one particle |
| $MSD_{bound}$ | 13 | 123 | 456 | 355 | one particle |
| $1SD_{bound} + MSD_{bound}$ | 73 | 180 | 490 | 379 | one particle |
| $1SD_{unbound}$ | 39 | | | | one particle + damping |
| $1SD_{unbound}$ | | 89 | 199 | 232 | two particles |

The two particles are thus, emitted either in a one-step or in a multistep reaction, respectively. As is mentioned in the introduction only simultaneous emission is important. For obvious reasons the 40 MeV limit is not sharp and coexistence of one-particle emission followed by damping and two-particle emission is likely to occur. Following the prescription of [5], the 1SD cross-section used as input to the calculation of the cross-section for the second neutron, 1SD2 (see fig. 3), is that of the (p,p′) reaction that excites both proton and neutron unbound particle-hole pairs.

The $1SD_{unbound}$ and the complementary 1SD2 cross-sections give an approximation of the 1SD two-particle spectrum of the theory of Ciangaru [6]. With increasing energy the integrated, total $1SD_{unbound}$ cross-section increases. However, the total $1SD_{unbound}$ remains practically constant with respect to the total one-particle emission cross-section of FKK (total $1SD_{bound}$ + $MSD_{bound}$ contributions), as is shown in table 2. The total one-particle emission of FKK amounts to approximately 65% (third row) of the total flux involved in the direct reactions, independent of the incident energy. The remaining 35% of the flux (fourth and fifth rows) is due to the $1SD_{unbound}$ and is mostly either damped or involved in the two-particle emission, depending on the incident energy. The finding that the two-particle, (p,np), emission comes up to 35% of the total flux, at energies higher than 40 MeV, is consistent with the corresponding results of 10–20% obtained for the Coulomb-suppressed (p,2p) knockout reaction [32].

One cannot expect a similar ratio between the two-particle and one-particle emissions for the proton inelastic scattering by iron shown in fig. 2. In the (p,n) reaction the final unbound states include a proton-particle and a neutron-hole. Thus, the two nucleons when emitted are of different type, *i.e.* the (p,np) reaction follows. On the other hand, since the (p,p′) reaction excites both proton- and neutron-particle-hole states, both the (p,2p) and the (p,pn) reactions contribute to the two-particle emission. However, the main difference is due to the isoscalar, collective enhancement of the $1SD_{bound}$ cross-section of the (p,p′) reaction, that appears approximately three times greater than the one for the (p,n) reaction. This makes the total one-particle emission of FKK dominating. In particular, for the case in fig. 2 the $1SD_{unbound}$ comes up to only 13%. Therefore the involved (compare table 1), quantitative analysis of the proton inelastic scattering by iron will be the subject of a separate, forthcoming report.

In figs. 1-3, the multiple evaporation from the compound nucleus, CN1, CP1, CNP1, . . . , CNP4, CN5, is calculated according to the theory of Hauser-Feshbach. The pre-equilibrium compound emission is important only at the low incident energy of 25 MeV, in fig. 1. It is calculated in the framework of the multistep compound (MSC) reaction theory of FKK [1,33], with account for the gradual absorption of incident flux [30,31]. The radial overlap integral of the single-particle wave functions in the MSC cross-section is calculated with constant bound-wave functions within the nuclear volume. The unbound wave is modified in accordance with the results of microscopic calculations [34].

## 3 Conclusions

The distinction between $1SD_{bound}$ and $1SD_{unbound}$ is important. Only the $1SD_{bound}$ cross-section describes well-defined one-step reactions that observe the EWSR limits, independent of the incident energy, and can be convoluted into the $MSD_{bound}$ cross-sections of FKK. Thus, the FKK theory describes only these direct processes that involve the emission of one particle. The more complicated processes are out of the scope of the FKK theory and are described by the $1SD_{unbound}$ to unbound particle-hole final states. The $1SD_{unbound}$ is evaluated by the approximate method [17]. At energies higher than 40 MeV $1SD_{unbound}$ can be treated in terms of the theory of knockout reactions to the continuum [6].

So far the particle binding restricted particle-hole states were used in the $Q$-chain of the multistep compound MSC reactions to describe the successive composite bound states embedded in the continuum before their decay into the exit channels [35]. We emphasize here for the first time that the particle binding restricted 1p1h states apply also to the continuum $P$-chain of the MSD reactions for the adequate description of the final bound particle-hole states at the MSD reaction stages.

It is also worth emphasizing that the distinction between the $1SD_{bound}$ and the $1SD_{unbound}$ cross-sections and the use of only $1SD_{bound}$ for folding into the $MSD_{bound}$ cross-sections is important even at low incident energies.

This is because the $1SD_{unbound}$ at low energies, although it does not contribute to the two-particle emission significantly, it allows for further damping transitions and therefore cannot be considered as a one-step cross-section.

The present paper completes the series of reports [2–4,15–18,36,27] that address the controversial issue of using DWBA or non-DWBA matrix elements [14,37]. We conclude that the enhanced, non-DWBA matrix elements provide $MSD_{bound}$ cross-sections for the one-particle emission, which together with the $1SD_{unbound}$ cross-section for the more complicated processes, describe well the emission of nucleons from nucleon-induced reactions all over the incident-energy range from 25 MeV to 120 MeV.

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